# A dive into the world of underwater waves: a.k.a: internal gravity waves

Earth Sciences Seminar

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*Thanks to: Prof. Stuart Dalziel, Prof. Andrew Lawrie*

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NERC DOCTORAL TRAINING

# What is fluid dynamics?

Fluid Dynamics is essentially how fluid moves:

- Ocean dynamics
- Volcanic Eruptions
- Climate and weather modelling
- Biological fluids
- Avalanche modelling
- Pipe lines and hydraulic modelling
- Flow over an aircraft wing
- Putting golden syrup on your pancakes

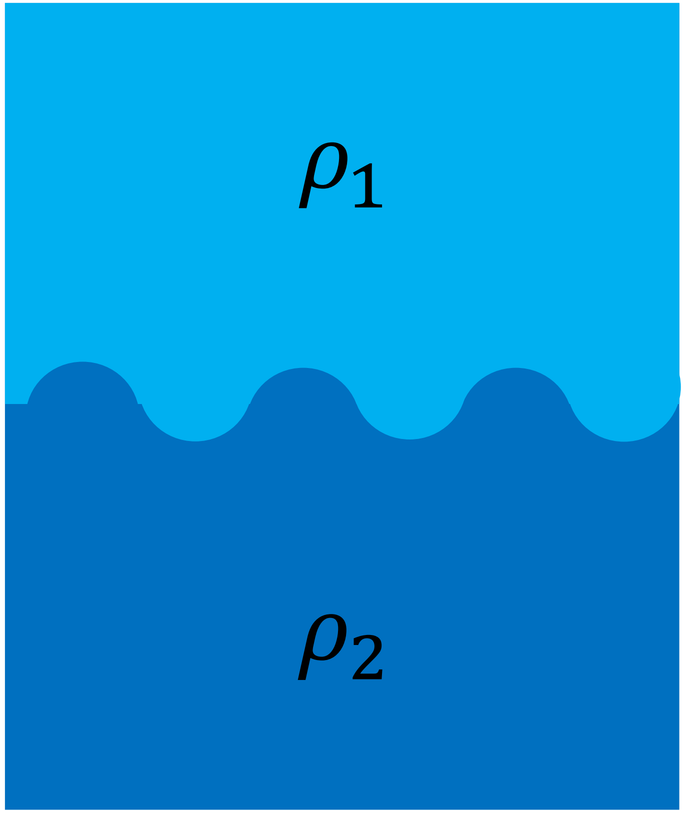
# Density differences

- Why does oil sit on top of water?
  - Due to density differences between the two fluids

Oil and water can be simplified to this:

- What happens if you perturb that interface?

$\rho_1 < \rho_2$



# Density stratification

An internal gravity wave is a wave that propagates **within** a fluid. Not on the surface. For this to happen it needs a stratification:

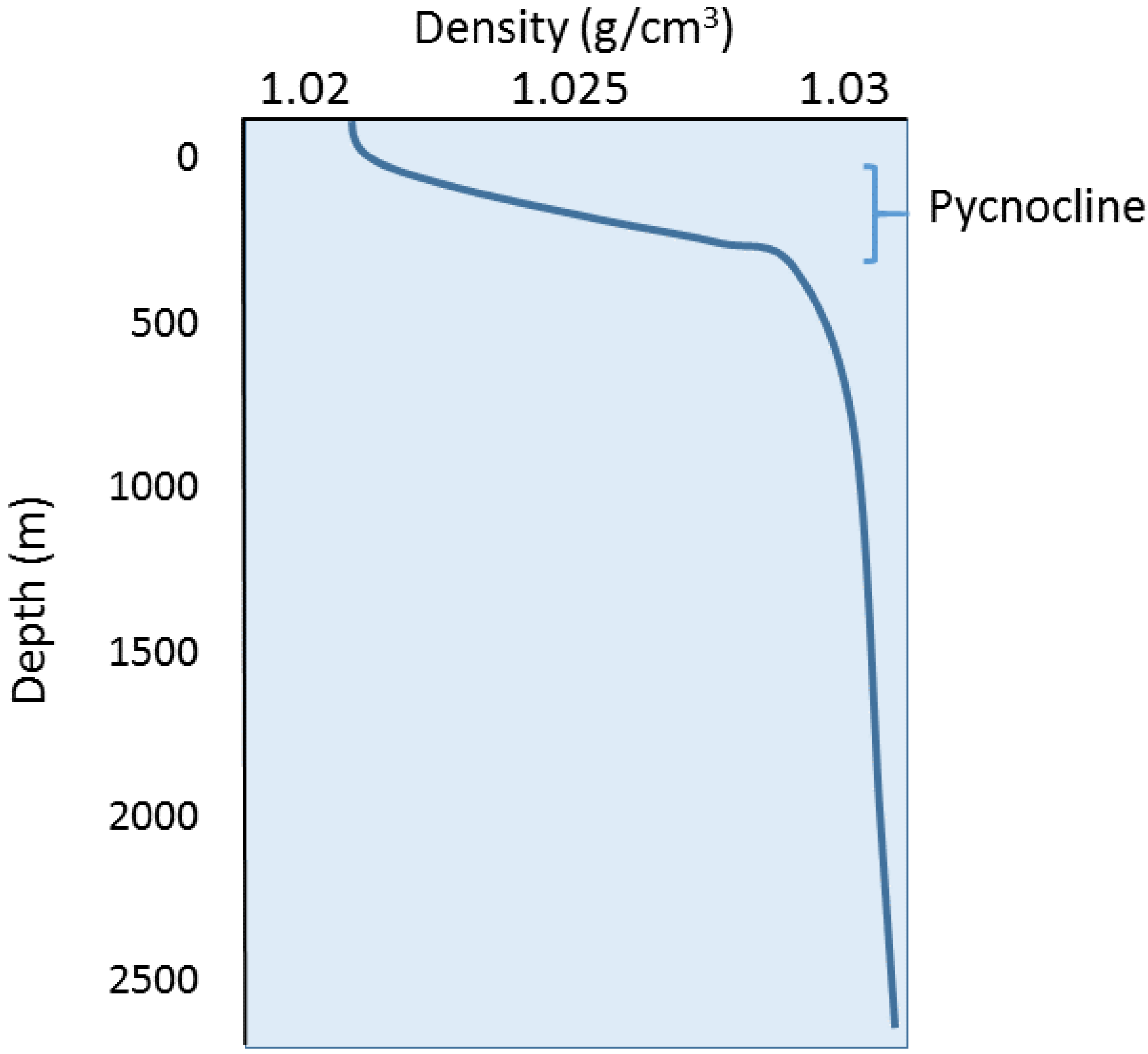
- **density changing with depth**

The buoyancy frequency $N$ governs the strength of this density change

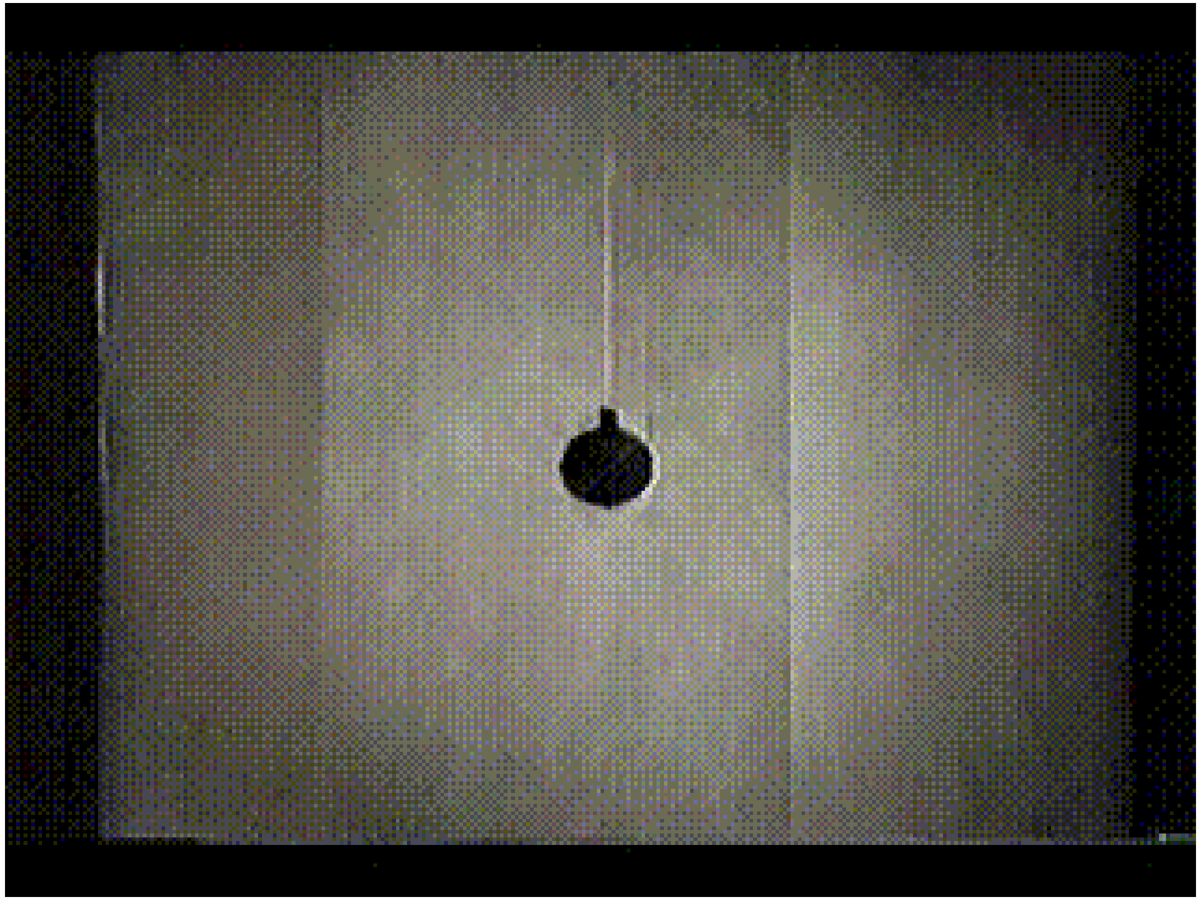
$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial z}}$$

If you displace a parcel of fluid up from it's neutral position, both a buoyancy force and an inertial force will be acting on and it will oscillate .

The ocean is stratified both temperature: allows for internal wave propagation.



# What are internal gravity waves?



**The famous 'Saint Andrews cross'**
Front view of a tank filled with a linear stratification

A cylinder oscillating vertically in a density stratification will produce four internal wave beams

Internal waves have the relationship that their energy propagation is perpendicular to their phase propagation.

This is opposite from surface waves which both point in the same direction.

Angle of propagation is given by:

$$\frac{\omega}{N} = \cos\theta$$

# Internal gravity waves in nature


*Image courtesy Jacques Descloitres, [MODIS Land Rapid Response Team](#) at NASA GSFC*
https://earthobservatory.nasa.gov/images/3586/internal-waves-sulu-sea

Paloma, Repsol-WhatsApp 🧀🐄🍪,
gravity waves over Barcelona

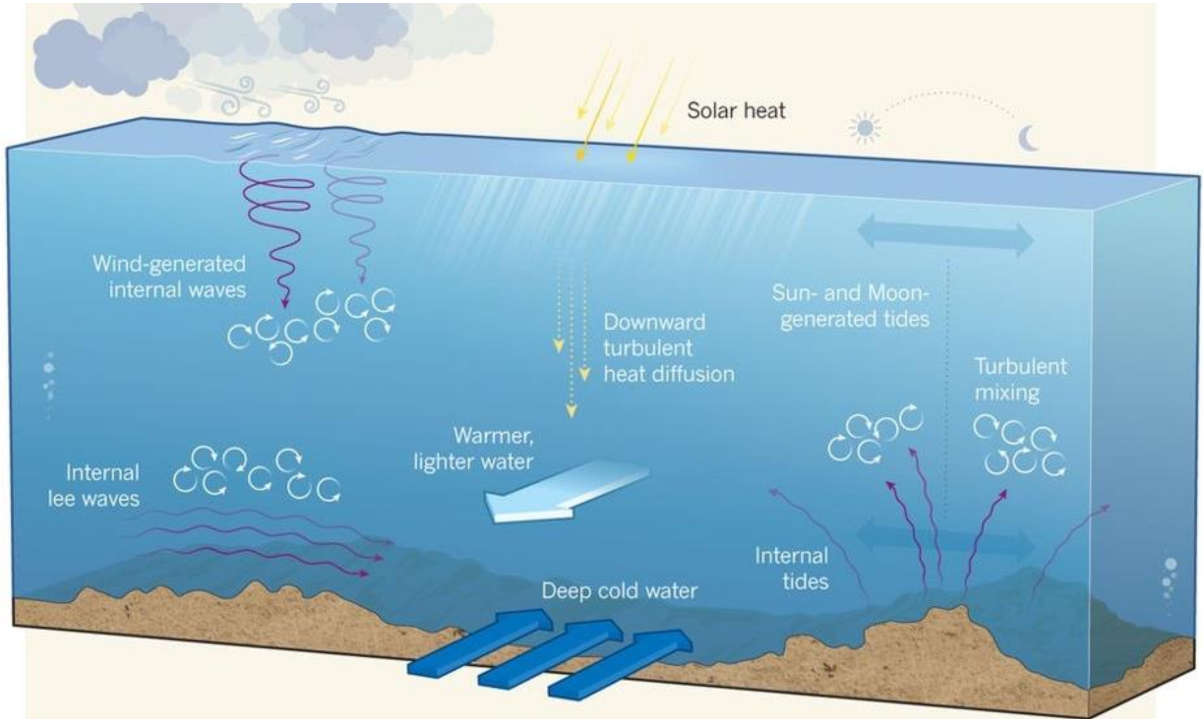
# Why do we care?

The main source of internal waves in the ocean are from tides oscillating over rough topography and wind.

- ocean circulation and climate models
- energy transport
- off shore construction

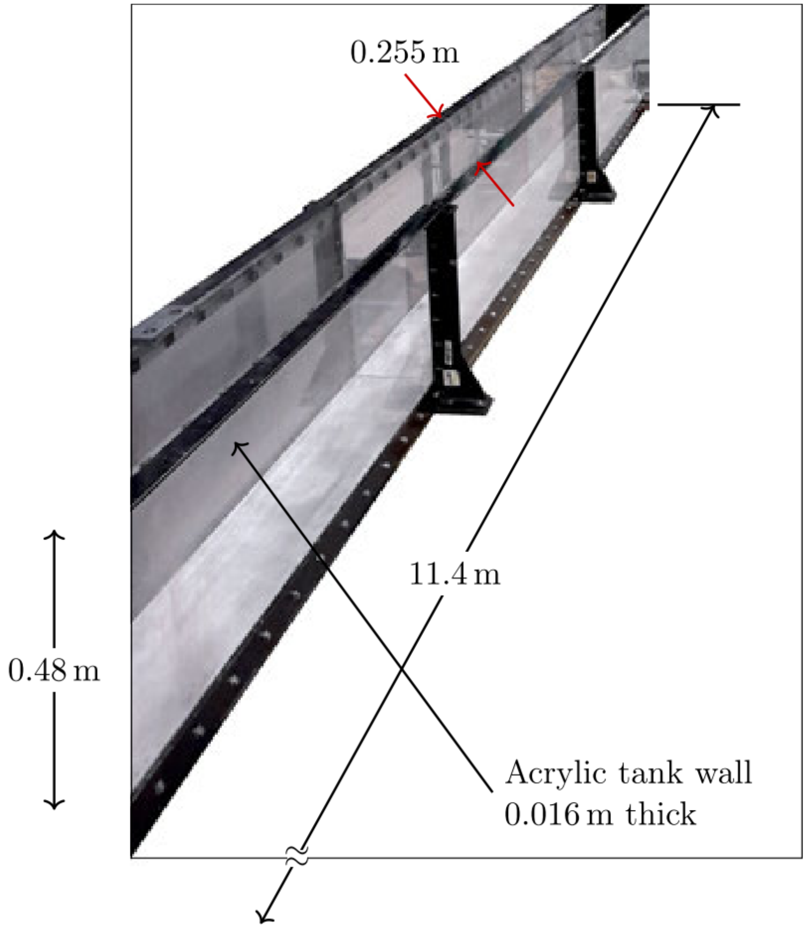
Contribute to 1TW of energy transport within the ocean* ~ 10 000 Olympic swimming pools of dense water per second.

How they break and deposit energy is not so well understood.




*Dobra, T. E. (2018). Nonlinear Interactions of Internal Gravity Waves, *24*(3), 493–536.
MacKinnon, J. (2013). Mountain waves in the deep ocean. *Nature*, *501*, 321. Retrieved from https://doi.org/10.1038/501321a

# Experimental setup

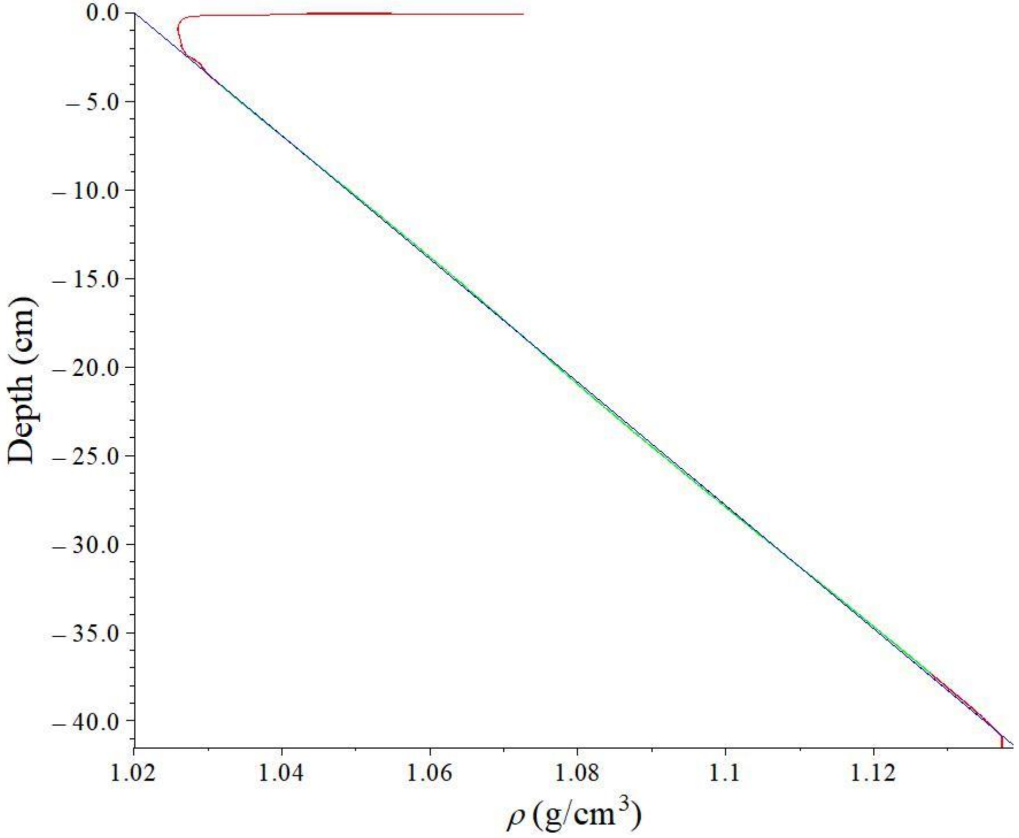



Dobra, T. E. (2018). Nonlinear Interactions of Internal Gravity Waves, *24*(3), 493–536.

- Perspex Tank
  0.26 m wide
  0.48 m deep
  11.4 m long

- The tank is filled with a linear stratification using two gear pumps.

- The stratification is measured using a conductivity probe.

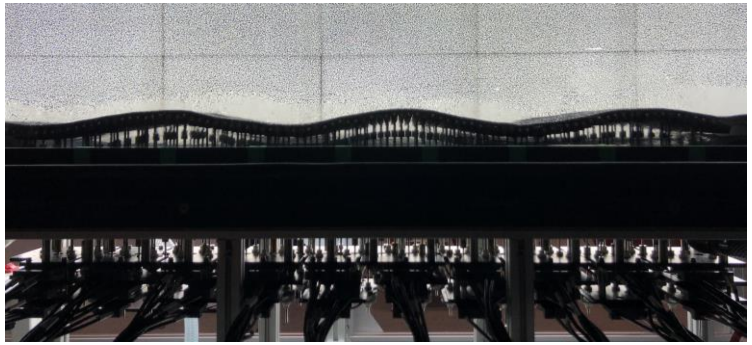


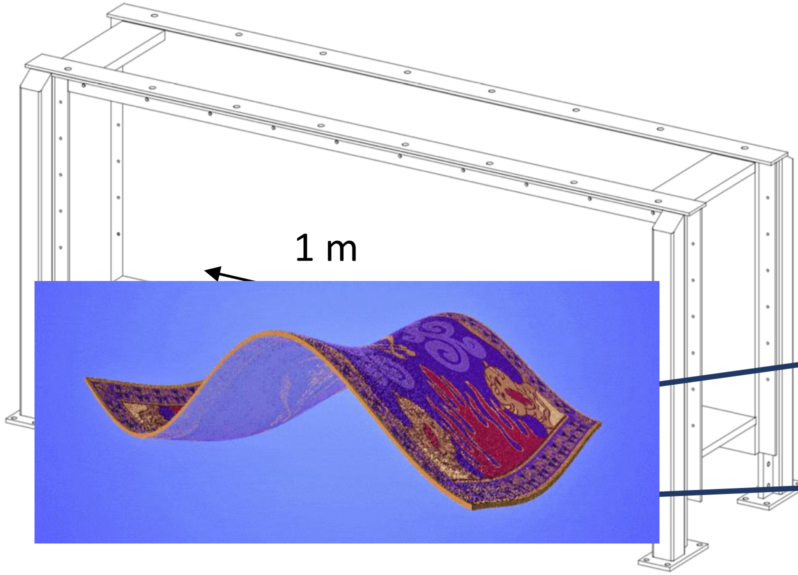

Example of the density stratification inside the tank measured with the conductivity probe.

# The wavemaker

To generate the internal waves we use a flexible bottom boundary called the Arbitrary Spectrum Wave Maker (ASWaM)

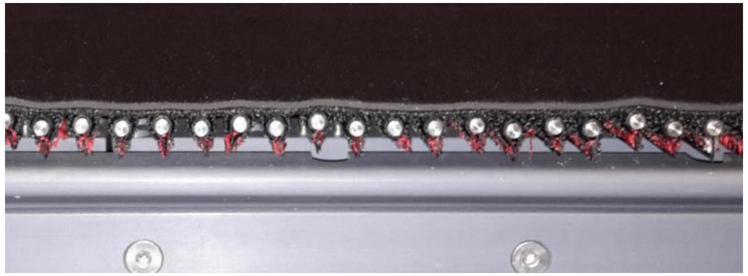
- 96 independently computer controlled horizontal rods
- Neoprene covering to interpolate between the rods
- Produces smooth, unidirectional beams with variable spatio-temporal forcing

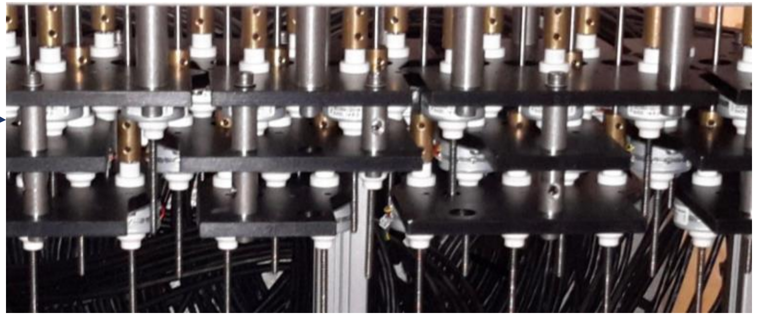





Horizontal rods

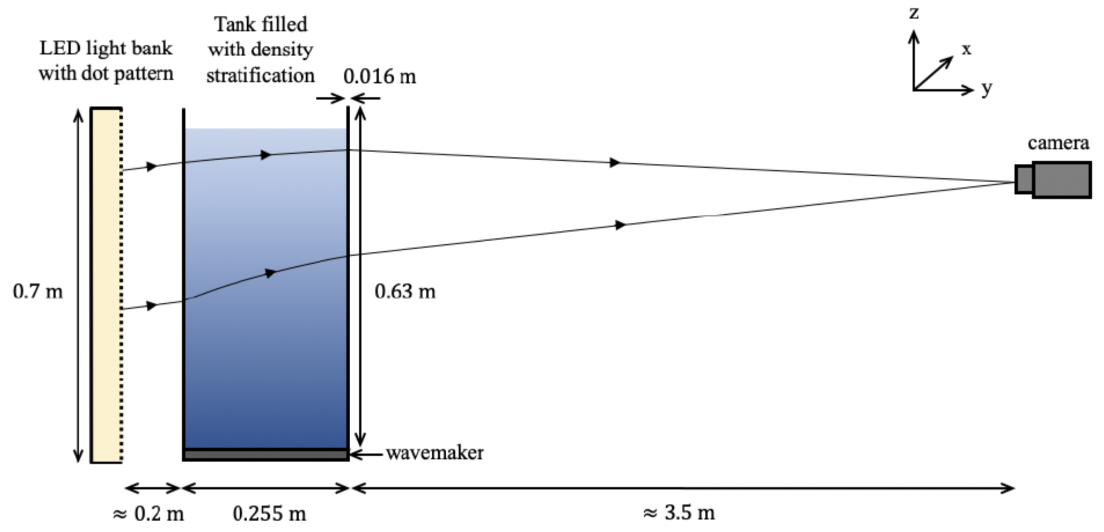
Motors





Dobra, T., Lawrie, A., & Dalziel, S. (2019). The magic carpet: an arbitrary spectrum wave maker for internal waves. *Experiments in Fluids*.

# Wavemaker in action




Side on view

Here the tank is filled with a linear stratification.

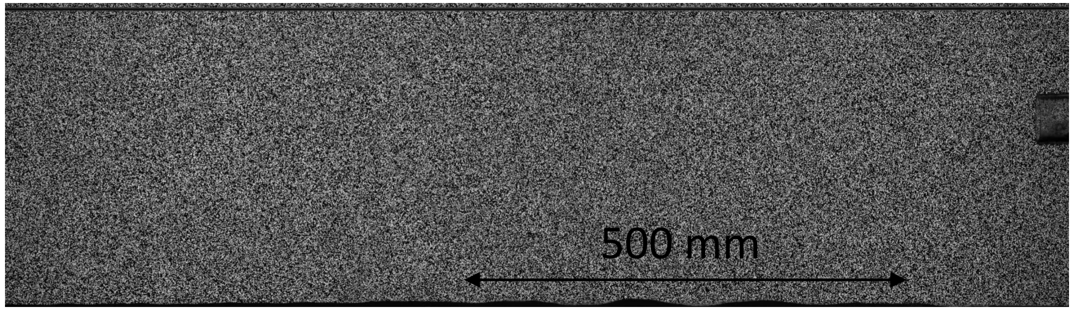
$N = 1.69$ rad $s^{-1}$

$\frac{\omega}{N} = 0.6$

$\eta_o = 4$ mm (forcing amplitude)

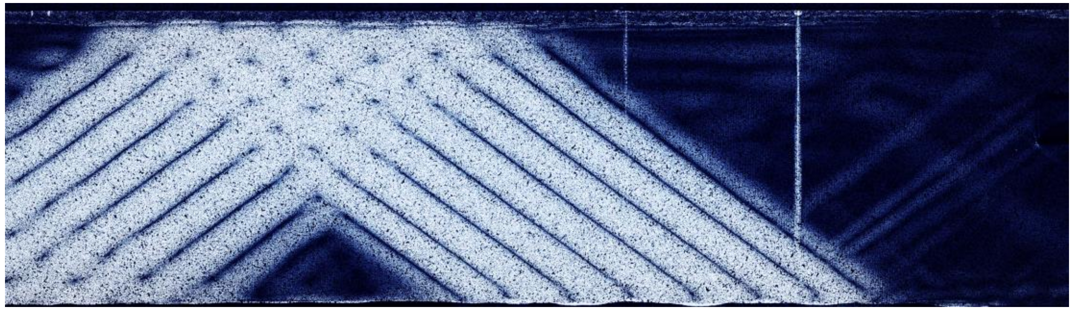
# Synthetic Schlieren

Raw Image



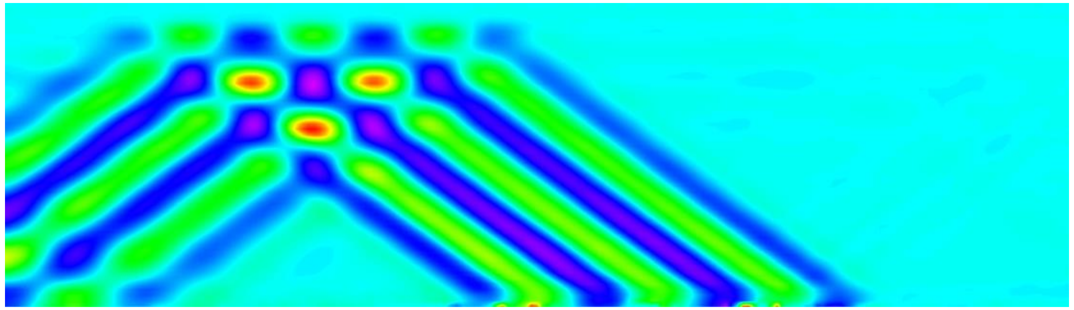

Qualitative Preview

The camera visualizes a side view of the tank.



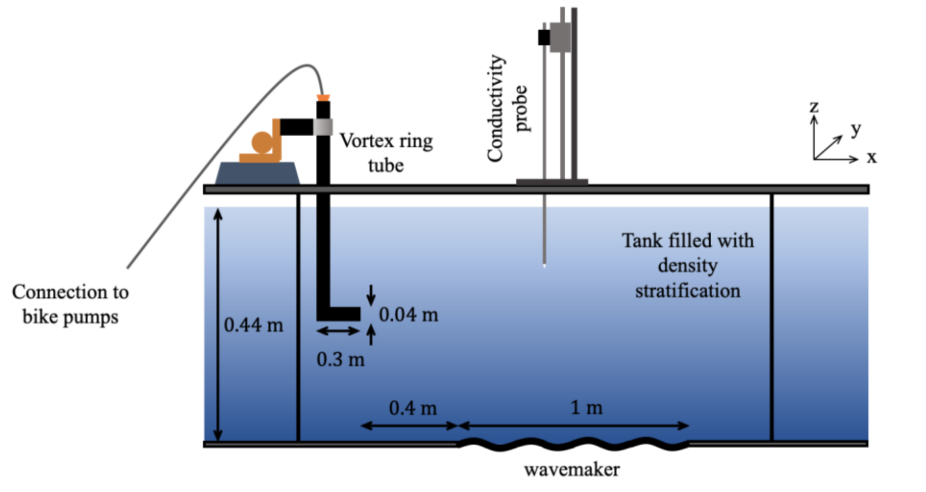
Processed Output

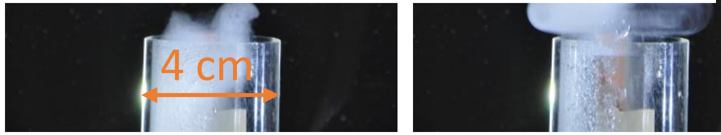
Once processed, Synthetic Schlieren measures the horizontal and vertical component of the gradient of the density perturbation

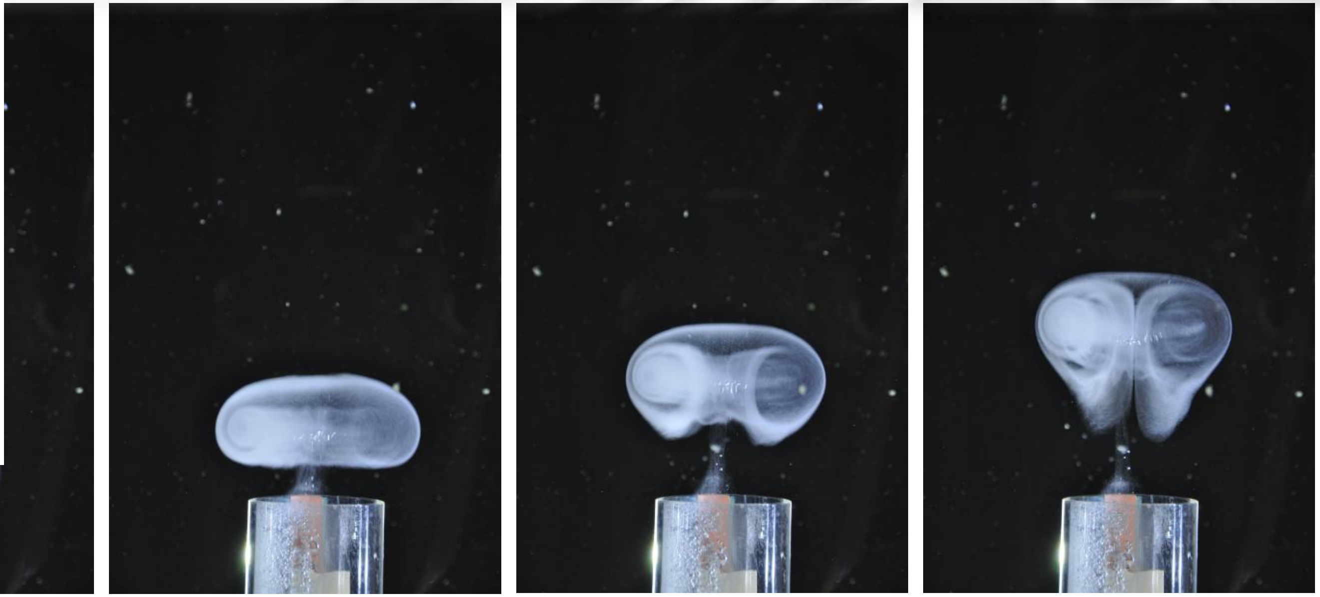


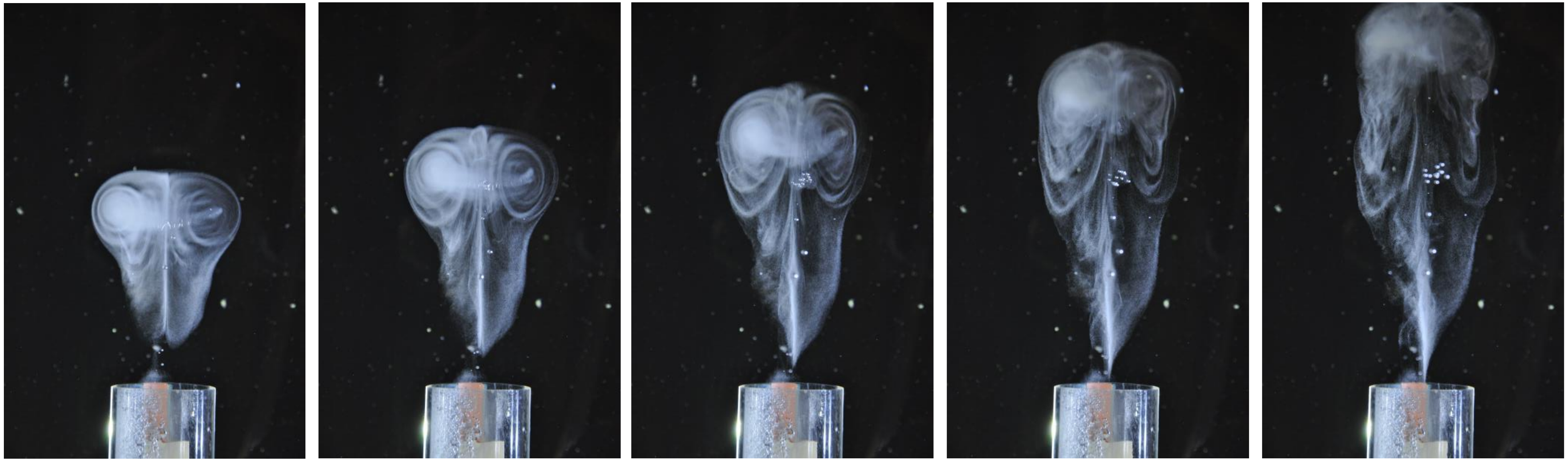
Output = $\beta_z = \frac{1}{\rho_0}\frac{\partial \rho'}{\partial z}$

# Vortex rings - plan view

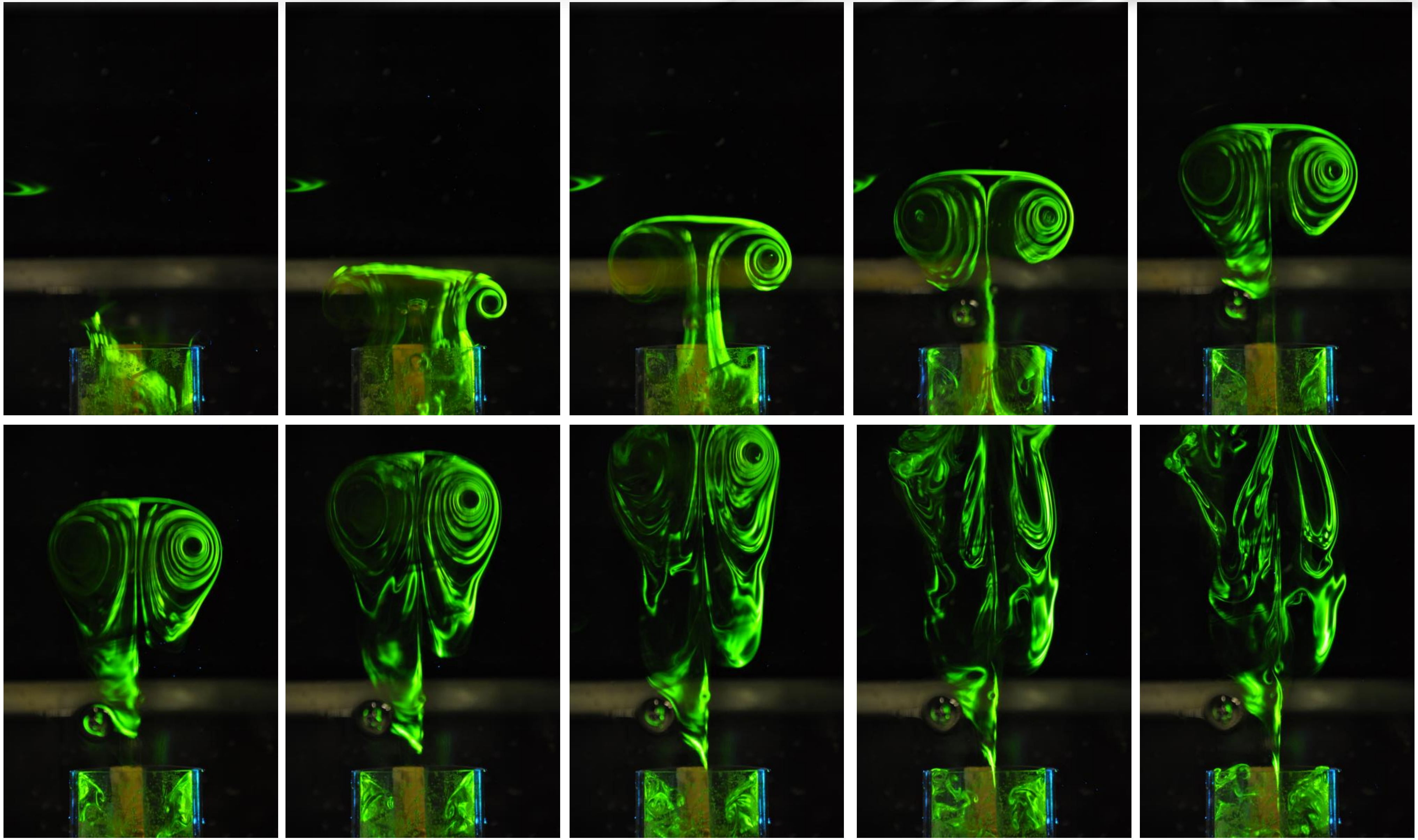




# Vortex rings - plan view



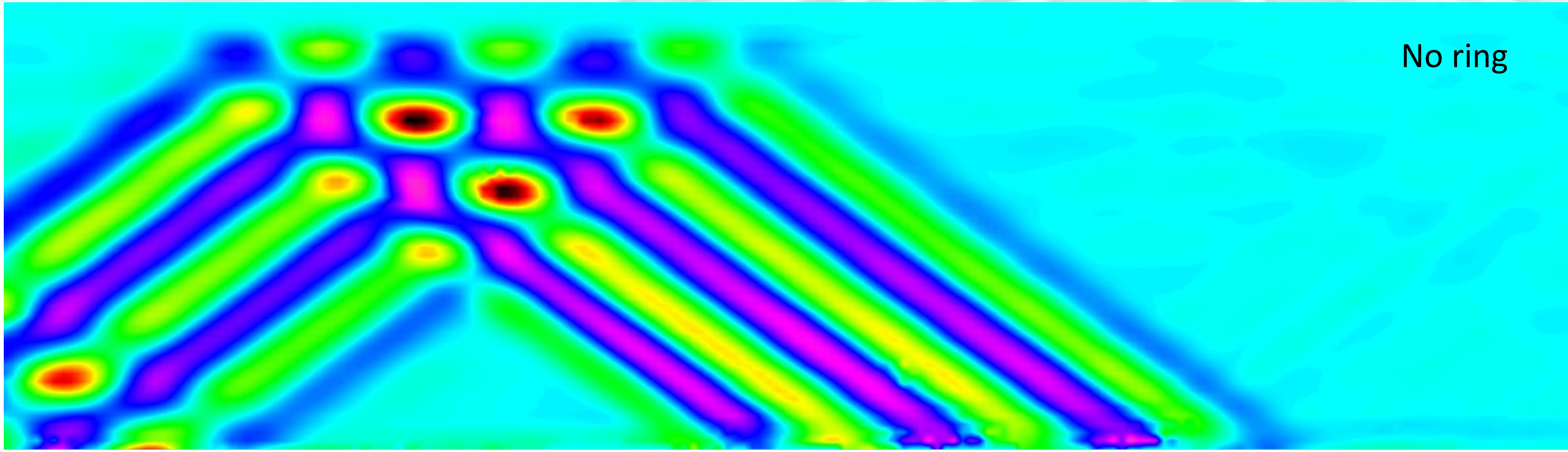
# Vortex ring / internal wave interaction

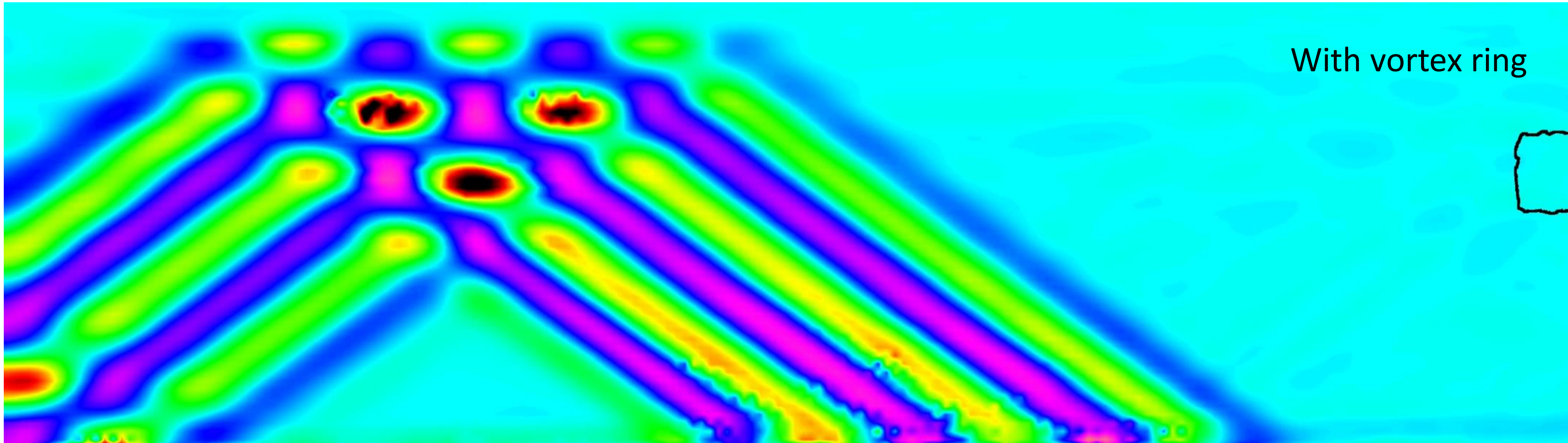
$N = 1.69$ rad $s^{-1}$

$\frac{\omega}{N} = 0.6$

$\eta_o = 4$ mm (forcing amplitude)

Video in real time



# What is Triadic Resonance Instability?

- Weakly non-linear resonant mechanism, also known as Parametric Subharmonic Instability (PSI)
- Primary wave becomes unstable to infinitesimal perturbations and generates the growth of two secondary waves
- Two secondary waves must satisfy:

$\omega_0 = \omega_1 + \omega_2$ (temporal resonant condition)

$\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$ (spatial resonant condition) $\boldsymbol{k} = (l, m)$

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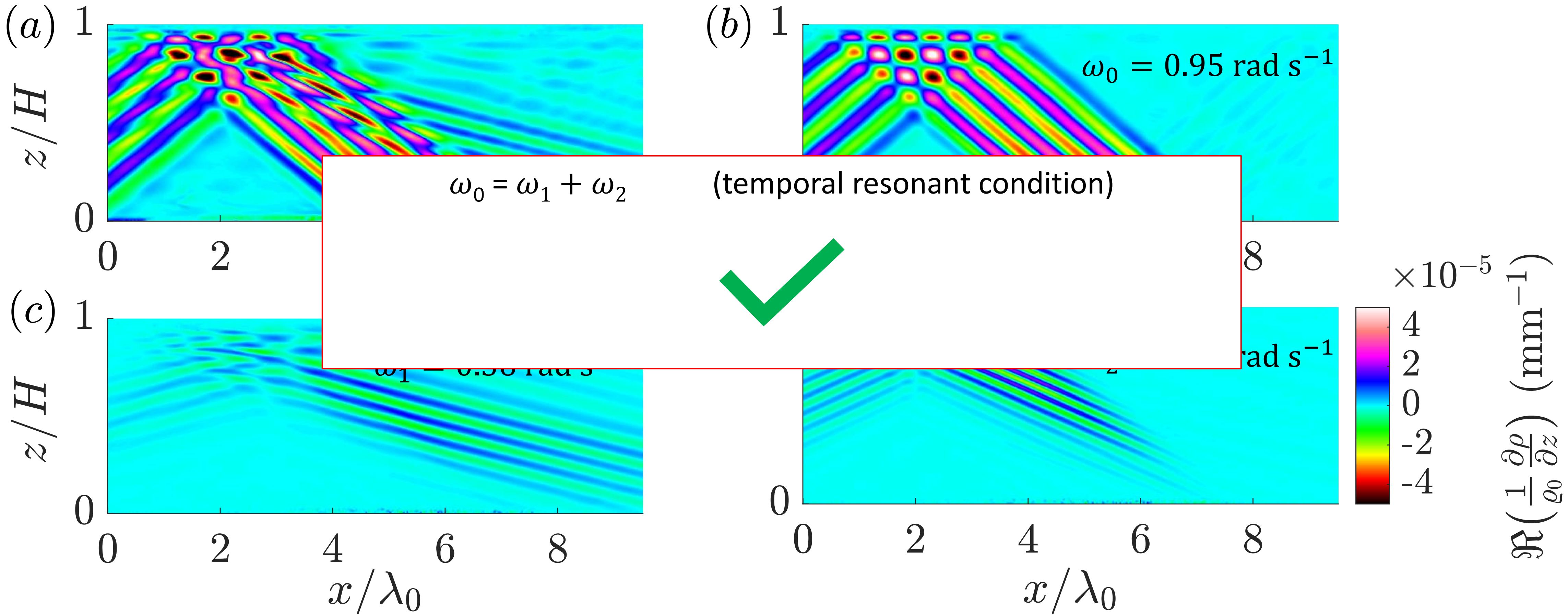
# Extraction of modes

- Use Dynamic Mode Decomposition (DMD) to find the recurrent 'modes' within the system

$$\omega_0 = \omega_1 + \omega_2$$

- Use the Hilbert Transform to find the direction of the phase velocity of these modes

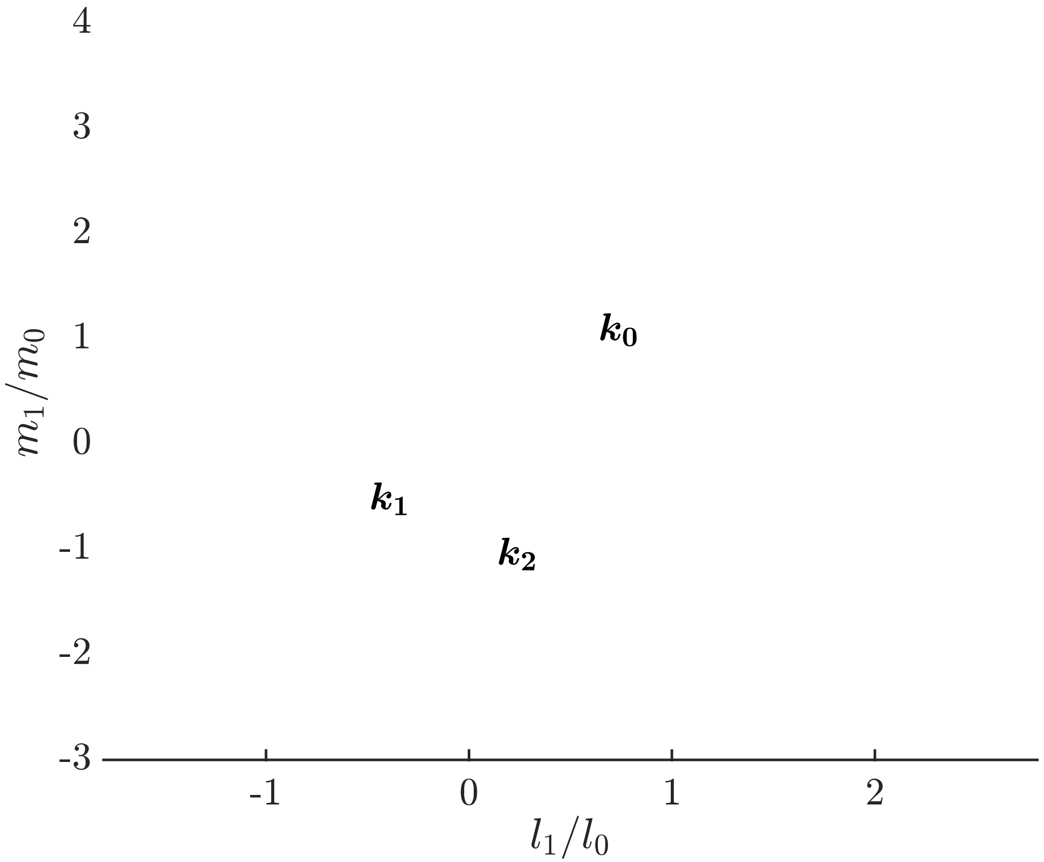



$\omega_0 = \omega_1 + \omega_2$ (temporal resonant condition)

# Do the experiments satisfy the conditions for TRI?

$\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$ (spatial resonant condition) $\boldsymbol{k} = (l, m)$

Blue arrows are showing experimentally generated wavenumber vectors.




Resonant loci curve showing all possible solutions $\boldsymbol{k_1}$ wave vector tip

# Conclusions

- Internal waves are buoyancy driven oscillations that occur inside a stratified fluid
- Crucial in understand how energy is transported in the global oceans
- Generated predominately by wind and tidal forcing
- Focused on what happens when you interact a vortex ring with an internal wave field
- The ring can act as a non-linear trigger for Triadic Resonance Instability